Lecture Notes for

Introduction to Probability and Statistics

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Probability and Statistics / 확률과 통계

강의노트 12

연속확률분포 교재정리

86. 정리

Definition 4.1.2 (Continuous density). Let X be a continuous random variable. A function f such that

1. $f(x) \geq 0$ for x real
2. $\int_{-\infty}^{\infty} f(x)dx = 1$
3. $P[a \leq X \leq b] = \int_a^b f(x)dx$ for a and b real

is called a density for X.

Necessary and Sufficient Conditions for a Function to be a Continuous Density

1. $f(x) \geq 0$
2. $\int_{-\infty}^{\infty} f(x)dx = 1$

87. 예제

Example 4.1.1. The lead concentration in gasoline currently ranges from .1 to .5 grams per liter. What is the probability that the lead concentration in a randomly selected liter of gasoline will lie between .2 and .3 grams inclusive? To answer this question, we need a density, f, for the random variable X, the number of grams of lead per liter of gasoline. Consider the function

$$f(x) = \begin{cases} 12.5x - 1.25 & .1 \leq x \leq .5 \\ 0 & \text{elsewhere} \end{cases}$$

88. cdf 정리

Definition 4.1.3 (Cumulative distribution—continuous). Let X be continuous with density f. The cumulative distribution function for X, denoted by F, is defined by

$$F(x) = P[X \leq x] \qquad x \text{ real}$$

Computing F Continuous Case

$$P[X \leq x] = F(x) = \int_{-\infty}^{x} f(t)dt \qquad x \text{ real}$$

89. 예제 - Example 4.1.2

Eample 4.1.1. 의 pdf 로 cdf 를 구하라.

$$P[X \leq x] = F(x) = \int_{-\infty}^{x} f(t)dt$$

$$F(x) = \int_{-\infty}^{x} f(t)dt = \int_{.1}^{x} (12.5t - 1.25)dt$$

$$= \left[\frac{12.5t^2}{2} - 1.25t\right]_{.1}^{x}$$

$$= 6.25x^2 - 1.25x + .0625$$

$$F(x) = \begin{cases} 0 & x < .1 \\ 6.25x^2 - 1.25x + .0625 & .1 \leq x \leq .5 \\ 1 & x > .5 \end{cases}$$

89. f (pdf) 와 F (cdf) 의 관계

f -> 적분 -> F
<- 미분 <-

90. Expectation and distribution parameters

Definition 4.2.1 (Expected value). Let X be a continuous random variable with density f. Let $H(X)$ be a random variable. The expected value of $H(X)$, denoted by $E[H(X)]$, is given by

$$E[H(X)] = \int_{-\infty}^{\infty} H(x)f(x)dx$$

provided

$$\int_{-\infty}^{\infty} |H(x)|\, f(x)dx$$

is finite.

Expected Value of X

$$E[X] = \int_{-\infty}^{\infty} xf(x)dx$$

91. 예제

Example 4.2.1. The density for X, the lead concentration in gasoline in grams per liter, is given by

$$f(x) = 12.5x - 1.25 \qquad .1 \le x \le .5$$

E(X), V(X), σ를 구하라.